Number Systems Study Material for Grade 10

1. Real Numbers

- **Definition**: Real numbers are a combination of rational and irrational numbers. They can be represented on a number line.
- **Rational Numbers**:
  - Definition: Numbers that can be expressed in the form $\frac{p}{q}$, where p and q are integers, and $q \neq 0$.
  - Examples: 2, $\frac{3}{5}$, -7, 0.
- **Irrational Numbers**:
  - Definition: Numbers that cannot be expressed as a ratio of integers.
  - Properties: Their decimal expansion is non-terminating and non-repeating.
  - Examples: $\sqrt{2}$, $\sqrt{3}$, π.
  - Proof Example: Prove that $\sqrt{2}$ is irrational:
    * Assume $\sqrt{2}$ is rational and can be expressed as $\frac{p}{q}$ in the lowest terms.
    * Then, $2 = \frac{p^2}{q^2} \implies p^2 = 2q^2$.
    * This implies p^2 is even, so p is even. Let $p = 2k$.
    * Substituting, $(2k)^2 = 2q^2 \implies 4k^2 = 2q^2 \implies q^2 = 2k^2$.
    * This implies q is even, contradicting the assumption that $\frac{p}{q}$ is in the lowest terms.

    Hence, $\sqrt{2}$ is irrational.

2. Properties of Real Numbers

- Closure, Commutative, Associative, Distributive, Identity, and Inverse properties as defined earlier.
- Example: Verify closure under addition for rational numbers.
  - Let $\frac{3}{4} + \frac{5}{6} = \frac{18}{24} + \frac{20}{24} = \frac{38}{24}$.
  - Since $\frac{38}{24}$ is rational, rational numbers are closed under addition.

3. Decimal Representation of Real Numbers

- Examples:
  - Terminating: $\frac{1}{4} = 0.25$, $\frac{7}{8} = 0.875$.
  - Non-Terminating Repeating: $\frac{1}{3} = 0.333\ldots$, $\frac{5}{6} = 0.8333\ldots$.
  - Non-Terminating Non-Repeating: $\pi = 3.141592\ldots$, $\sqrt{2} = 1.41421\ldots$.

4. Prime Factorization and LCM/HCF

- **Prime Factorization**:
  - Breaking a number into its prime factors.
  - Example: $60 = 2^2 \cdot 3 \cdot 5$.
- **LCM (Least Common Multiple)**:
  - Definition: The smallest number divisible by all given numbers.
  - Steps:
    - Perform prime factorization of all numbers.
    - Take the highest power of all prime factors.
  - Example: Find the LCM of 12 and 18.
    - Prime factorization: $12 = 2^2 \cdot 3$, $18 = 2 \cdot 3^2$.
    - LCM: $2^2 \cdot 3^2 = 36$.
- **HCF (Highest Common Factor)**:
  - Definition: The largest number that divides all given numbers.
  - Steps:
    - Perform prime factorization of all numbers.
    - Take the lowest power of common prime factors.
  - Example: Find the HCF of 12 and 18.
    - Prime factorization: $12 = 2^2 \cdot 3$, $18 = 2 \cdot 3^2$.
    - HCF: $2 \cdot 3 = 6$.
- **Relation Between LCM and HCF**:
  - $LCM \cdot HCF =$ Product of Numbers.
  - Example: For 12 and 18, $LCM = 36$, $HCF = 6$.
  - Verify: $36 \cdot 6 = 12 \cdot 18$.

5. Rationalization

- Example: Rationalize $\frac{1}{\sqrt{2}}$.
  - Multiply numerator and denominator by $\sqrt{2}$.
  - Result: $\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$.
- Example: Rationalize $\frac{3}{\sqrt{5}-\sqrt{3}}$.
  - Multiply numerator and denominator by the conjugate: $\sqrt{5} + \sqrt{3}$.
  - Result: $\frac{3(\sqrt{5}+\sqrt{3})}{5-3} = \frac{3\sqrt{5}+3\sqrt{3}}{2}$.

6. Laws of Exponents for Real Numbers

- Example:
  - Simplify $(2^3)^2 \cdot 2^{-4}$.
  - Solution: $(2^3)^2 = 2^6$, $2^6 \cdot 2^{-4} = 2^{6-4} = 2^2 = 4$.

7. Surds

- Example: Simplify $\sqrt{50} + \sqrt{72}$.
  - Simplify: $\sqrt{50} = 5\sqrt{2}$, $\sqrt{72} = 6\sqrt{2}$.
  - Result: $5\sqrt{2} + 6\sqrt{2} = 11\sqrt{2}$.

8. Examples and Practice Problems

- Calculate the LCM and HCF of 20 and 30.
- Prove $\sqrt{3}$ is irrational.
- Simplify $\frac{\sqrt{5}}{\sqrt{2}+1}$.

9. Key Tips for Problem Solving

- Always simplify the expression to its lowest terms.
- Identify patterns in the problem (e.g., terminating or non-terminating decimals).
- Cross-verify calculations, especially for LCM and HCF.